Two-Dimensional Heat Transfer Through Complex-Shaped Composite Materials—Part II: Verification and Results

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The development of the model simulating two-dimensional heat transfer through complex-shaped composite materials including a heat generation term and exhibiting boundary movement was outlined in Part I of this paper. This part includes the code verification. Temperature profiles predicted using the code are compared to results from analytical solutions and finite element solutions as well as experimentally determined temperatures from autoclave and press cures of composite materials. Additional experimental data are needed to verify the modeling of the effect of compaction on heat transfer and to further test the modeling of heat transfer in complex shapes.

Nomenclature

G	= heat generation per unit volume, W/m^3
H	= height, m
k	= thermal conductivity, W/m-K
L	= length, m
T	= temperature, K
T_∞	= autoclave temperature, K
T_i	= initial temperature, K
t	= time, s
x,y,z	= Cartesian coordinates
α	= degree of cure or extent of reaction
α_T	= thermal diffusivity, m^2/s
β_m	= eigenvalues of analytical solution, $m\pi/L$
θ	= modified temperature, $T - T_i$, K
μ	= dynamic viscosity, kg/m-s
ν_n	= eigenvalues of analytical solution, $n\pi/H$
ρ	= density, kg/m^3
$\phi(x)$	= boundary condition function of analytical solution
ϕ_0	= slope of boundary condition function

Homogeneous Materials

Rib Shape

In order to illustrate the model's ability to accommodate complex geometries, the temperature distributions were predicted for the rib shape system shown in Fig. 1. The physical and thermal properties for this case are presented in Table 1. This shape is symmetric only across the centerline. The temperature profile should reflect this symmetry in the x direction. Indeed, for both temperature and convection boundaries, the results did show the symmetry. The solution is also reasonable since the constant x temperature profiles show higher temperatures at higher values of y. This is because as y increases, the width decreases, thus decreasing the resistance to heat transfer and resulting in higher temperatures.

Chimney Problem

A steady-state solution for a complex shape system has been obtained using the developed code. The results have been compared to a solution generated using a finite element analysis.[1] The original problem was to determine the temperature distribution inside the 3.1 × 1.8 m chimney wall. Two circular stacks with a diameter of 0.61 m are centered in the wall. The stack wall is maintained at 438.7 K and the outer wall at 327.2 K. Because of the symmetry of the geometry, the temperature distribution is generated in only half of the system, with the temperature contours in the other half being symmetric. The thermal conductivity for the chimney wall is 2.84 W/(m-K). The discretized domains obtained using the body-fitted coordinate (BFC) system generation technique are shown in Fig. 2. The grid shown in Fig. 2a was generated without attraction of

Table 1 Physical and thermal properties of the homogeneous material

Width, m	0.1
Height, m	0.1
Number of layers	3
Thermal conductivity, W/(m-K)	0.05
Heat capacity, J/(kg-K)	800
Density, kg/m^3	1000
Time step, s	100
Number of nodes	21
Heat transfer coefficient, W/(m^2-K)	200

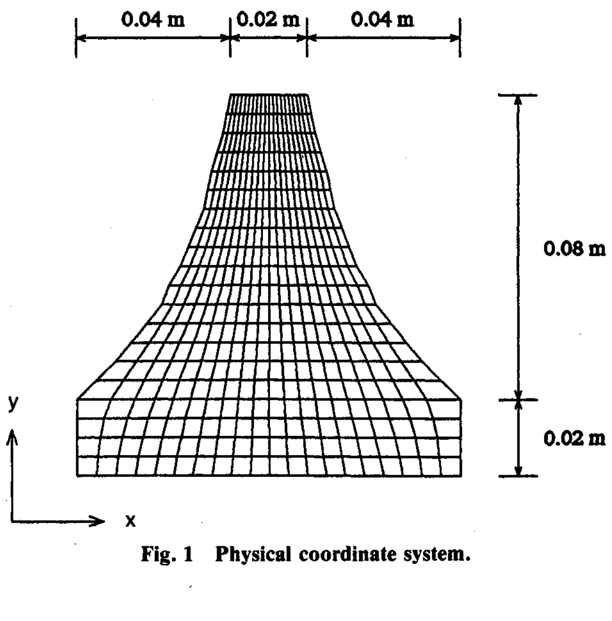

Fig. 1 Physical coordinate system.

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Table 2 Analytical solution of two-dimensional transient heat transfer in rectangular systems

Constant boundary conditions:

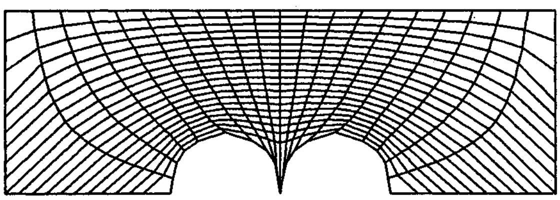
$$\theta(x,y,t) = \frac{4}{HL} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\nu_n}{\beta_m^2 + \nu_n^2} \sin\beta_m x \sin\nu_n y \times [1 - e^{-\alpha_T(\beta_m^2 + \nu_n^2)t}] \frac{\phi}{\beta_m} (1 - \cos\beta_m L) \right\}$$

Variable boundary conditions:

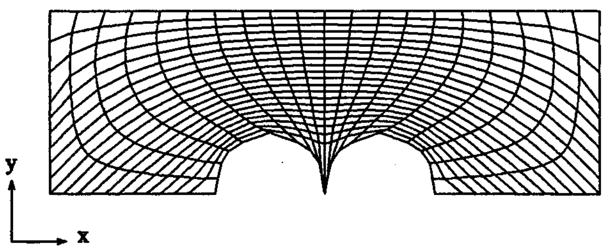
$$\theta(x,y,t) = \frac{4}{HL} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\nu_n}{\beta_m^2 + \nu_n^2} \sin\beta_m x \sin\nu_n y \times [1 - e^{-\alpha_T(\beta_m^2 + \nu_n^2)t}] \phi_0 \left(\frac{1}{\beta_m^2} \sin\beta_m L + \frac{x}{\beta_m} [1 - \cos(\beta_m L)] \right)$$

Solution with heat generation term:

$$\theta(x,y,t) = \frac{4G(t)}{HLk} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{1}{\nu_n \beta_m (\beta_m^2 + \nu_n^2)} \sin\beta_m x \sin\nu_n y \times [1 - e^{-\alpha_T(\beta_m^2 + \nu_n^2)t}] (1 - \cos\beta_m L)(1 - \cos\nu_n H) \right\}$$



a) BFC discretization without attraction



b) BFC discretization with attraction

Fig. 2 Discretized domain for chimney.

grid lines to any point in the domain. Because of a deficiency of nodal points near the bottom two corners, the attraction parameters were reset in order to generate a grid with a more suitable nodal distribution. The resulting grid with attraction to the bottom corners is shown in Fig. 2b. Note that the ability to redistribute grid points and control grid point location and density by varying input control parameters is a useful feature of the BFC grid generation technique. The isotherms for each solution are shown in Fig. 3. The temperature isotherms obtained using the proposed model solution show agreement with those of the finite element method. It should be noted, however, that the isotherms of the finite element method are average temperatures of each element, and those of this model are nodal temperatures.

Analytical Solutions

An analytical solution has been programmed using the integral transform method as outlined by Özisik[2] in order to solve the two-dimensional transient energy equation in rectangular coordinates for a system of constant properties. This analytical solution was used to verify several features of the two-dimensional heat transfer code.

The analytical method consists of transforming the partial derivatives into total derivatives using one-dimensional integral transforms. The resulting ordinary differential equation is then solved, and the temperature is found by successively inverting the multiple transforms of the temperature. Three analytical solutions have been developed for two systems. The first is a 1.0×0.3 m rectangle and the second is a 0.153×0.0285 m rectangle. The second system is used to simulate dimensions of a 0.153×0.153 m, 128-ply "prepreg" with a 48-ply bleeder and a temperature that it is subjected to during its cure. The first solution accommodates a constant temperature boundary condition of 400 K, an initial condition of 300 K, and no heat generation. The second involves an initial condition of 300 K, with top, left, and right boundaries at a temperature of 300 K and a boundary condition that is varying linearly with the distance along the bottom edge ($300 + 100x$ K). The third solution uses a 300 K initial temperature, a 400 K temperature at all four boundaries, and includes a heat generation term, which varies exponentially as a function of time ($G = 100 \exp[0.0092t]$). Each of these methods is designed to isolate and verify one feature of the numerical solution. The first allows the verification of the differencing scheme. The second is used to illustrate the flexibility of the code and to verify the ability to accommodate variable boundary conditions. The third solution is set up to verify the effect of heat generation and the simulation of the

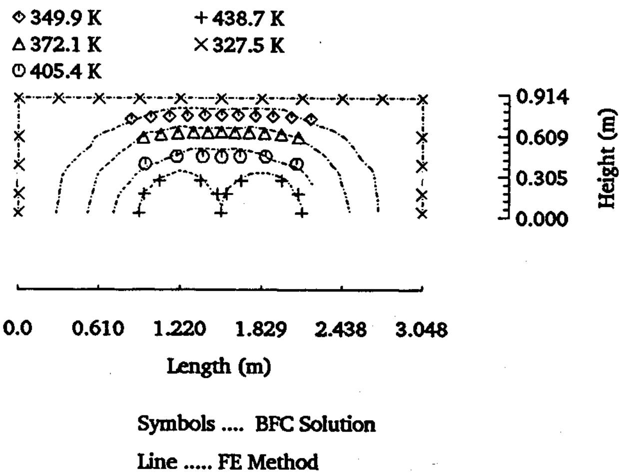


Fig. 3 Isotherms in chimney from BFC and finite element methods.

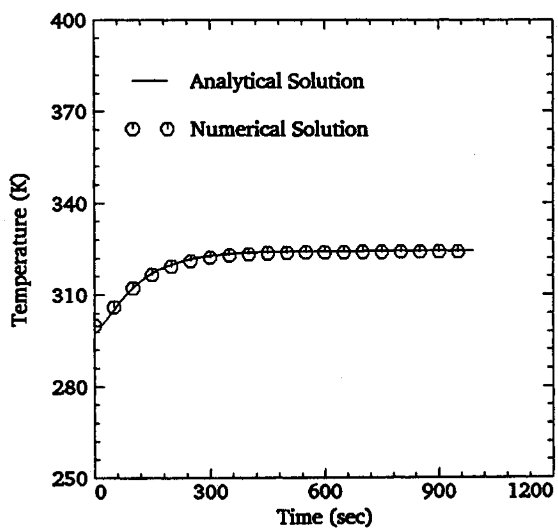


Fig. 4 Analytical and numerical solutions for system I with variable boundary conditions.

exothermic chemical reactions taking place during the cure. The expressions for all three solutions are listed in Table 2.

The results of the numerical solution show agreement with the results of the analytical technique for all four of the cases just outlined. Only the cases for the heat generation term (dimensions of system 2, temperature at center) and the case of a variable boundary condition (dimensions of system 1, temperature at center) are shown in Figs. 4 and 5. Although the cases checked are for stationary boundaries with constant properties, agreement between the numerical and analytical

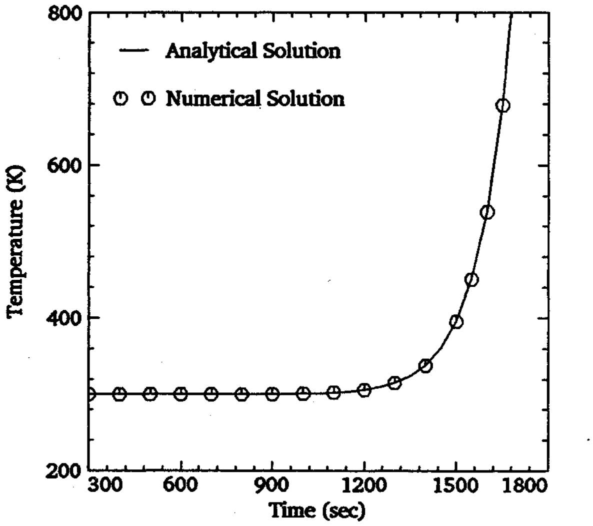


Fig. 5 Analytical and numerical solutions for system II with heat generation.

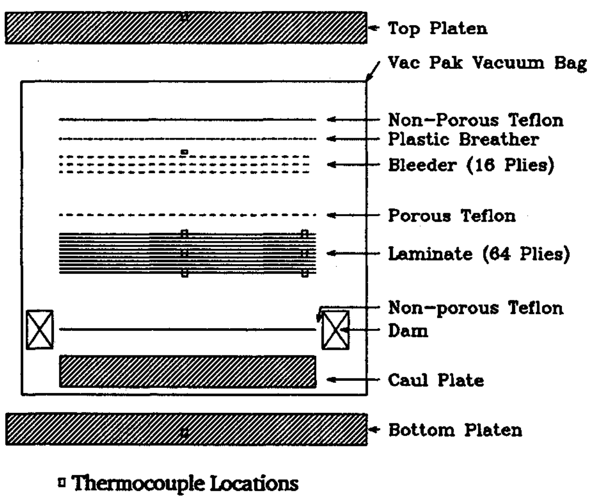


Fig. 6 Layup of TEST1.

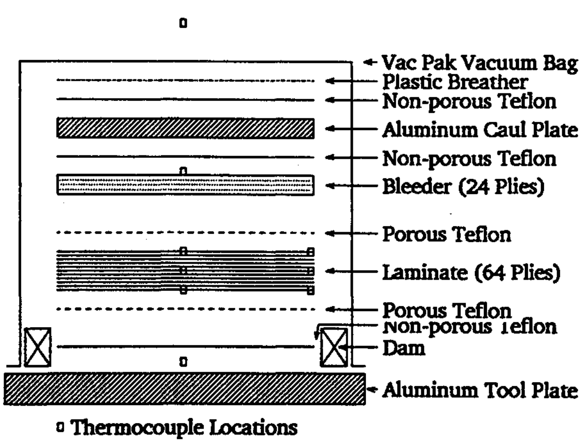


Fig. 7 Autoclave layup of TEST2.

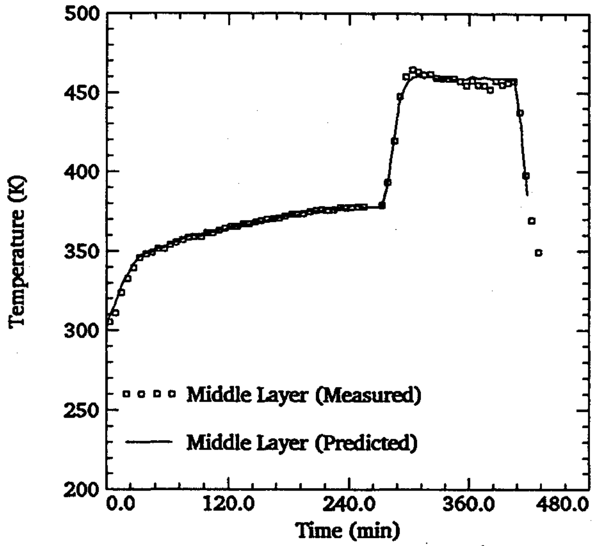


Fig. 8 Predicted vs measured temperature in composite press cure.

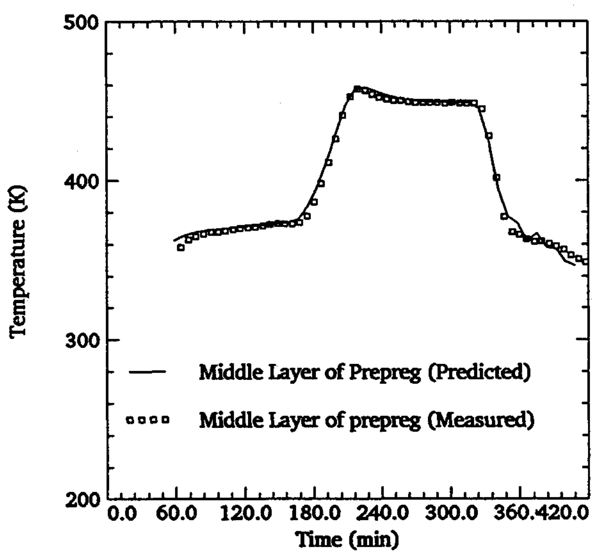


Fig. 9 Measured vs predicted temperature with convection boundary condition.

solutions builds confidence in the differencing scheme, in the ability to manage variable boundary conditions, as well as accurately representing heat generation terms.

Autoclave and Press Cure Modeling

The two-dimensional heat transfer model was used to predict temperature distribution in thermosetting composites during cure. In this section, the predicted results are compared to experimental data.

Experimental Runs

Three experimental runs were used to monitor the temperature distribution during the cure of Hercules AS4/3501-6. These runs are referred to as TEST1–TEST3. TEST1 was a press cure of a prepreg panel, and TEST2 and TEST3 were obtained during autoclave cure. The layups and thermocouple locations for TEST1 and TEST2 are presented in Figs. 6 and 7. The layup in TEST3 consisted of the same bagging materials as those of TEST2 with a 128-ply prepreg and 48 plies of bleeder. Experimentally measured temperatures at the boundaries or in surrounding autoclave air were used as boundary conditions. The predicted temperatures in the interior of the composite were then compared to measured temperature distributions.

Press Cure

TEST1 represents a press cure run of a 64-ply prepreg. Unfortunately, only thermocouples around the prepreg were functional. Therefore, the temperature in the platens is not available for use as a boundary condition. The temperatures around the prepreg were used to predict the temperature in the interior of the prepreg. The results presented in Fig. 8 show agreement with experimental results. Experimental data from other press runs show agreement between measured and predicted temperatures at various positions (in the x and y directions) within the composite and the bagging materials.

Convective Boundary

The ability to handle convective boundary conditions is essential in order for the model of the cure cycle to be of practical use in a manufacturing environment. In this case, only the autoclave temperature as a function of time would be required. The need to measure temperatures inside the bagging material, which then requires the placement of thermocouples in the layup, would be eliminated.

Although a constant value of the heat transfer coefficient characteristic of a forced convection was used for the results presented here, a correlation relating the heat transfer coefficient to temperature, pressure, and type of convective medium is now available.[3] The autoclave temperature and conditions of TEST3 were utilized to predict the temperature distribution inside the material. Agreement between predicted and measured temperatures was observed throughout the cure cycle (for all thermocouple locations). Only the temperature at the top and middle layers of the prepreg are shown in Figs. 9 and 10.

Importance of Heat Transfer

Heat transfer is the dominant phenomena since it allows the prediction of the temperature distribution inside the material. The temperature is the most important parameter of the overall modeling process because most properties are dependent on it. Once the temperature distribution has been developed, properties such as viscosity and degree of cure can be calculated from previously established correlations.[4] Figure 11 shows the variations of such properties with the temperature. These established and verified correlations can be used to calculate the viscosity and degree of cure at various points in the cure cycle when in-situ sensors are not available.

Other Model Features

Several boundary conditions can be used in this code, which increases the applicability of the heat transfer model to a variety of processes. Variable boundary conditions dependent on time and position and including specified temperature, specified heat flux, and convective and insulated boundaries can be accommodated. For the time and spatial step sizes used, run time is less than real time, which makes the code very attractive for future process control applications. Finally, the code's flexible structure permits the incorporation of other submodels such as kinetic, flow, or stress submodels without altering the remainder of the code.

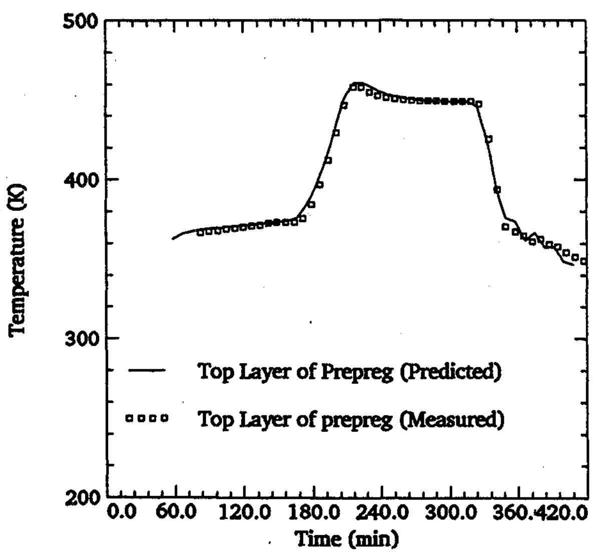


Fig. 10 Measured vs predicted temperature with convection boundary condition.

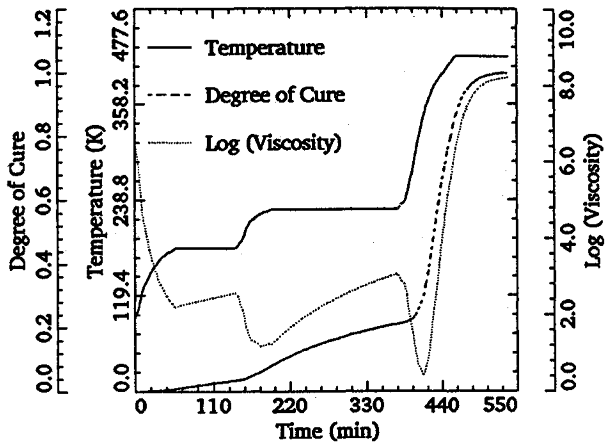


Fig. 11 Viscosity and degree of cure as a function of temperature.

Summary

The energy equation describing the two-dimensional transient heat transfer through complex-shaped, heterogeneous composite materials with variable properties and including an internal heat source was solved using the body-fitted coordinate system generation and implicit finite-difference method. Constant and time-dependent boundary conditions can be accommodated.

The code has been successfully verified using analytical solutions for systems with constant properties, constant and variable boundary conditions, and a heat generation term. The ability to accommodate complex shapes was verified by comparing the results with finite element solution for a complex-shaped system with constant properties. The model was also verified using experimental results from autoclave and press curing of graphite/epoxy prepregs for both temperature-specified and convective-boundary conditions.

An automatic grid generation code was also developed. This body-fitted coordinate system generation technique allows for the study of heat transfer through complex-shaped materials and accounts for surface movement due to compaction. The grid generation code is general, can be used for any system, and allows for automatic control of grid nodes location and density.

Future Work

Additional experiments are needed to test existing parts of the code. Autoclave curing of complex shapes is necessary to test the complex two-dimensionality of the problem further. A general correlation[3] relating the heat transfer coefficient to temperature, pressure, and convective medium needs to be incorporated in the model. Compaction rate measurements are needed to test the ability to accommodate boundary movement and its effect on heat transfer. The energy equation should be modified, and cross terms of the thermal conductivity tensor should be evaluated in order to extend the applicability of the model to any angle ply laminate (stacking sequence with fiber direction not along the x or y axis).

Several aspects of the overall problem are still under development. The property data base is being extended to include other resin systems as well as the dependence of these properties on prevailing conditions. Kinetic rates for other thermosets are also needed in order to increase the applicability of the code to modeling other prepregs.

The pressure/flow model already developed[5] needs to be incorporated in the model. Parameters predicted by such a model are resin flow rate, pressure distribution, and compaction rate, which is an essential parameter in treating boundary movements. The flow need not be coupled with the energy equation for the autoclave and press cure of composites. Coupling of the mass, momentum, and energy balances is necessary, however, when modeling other processes such as pultrusion.

The two-dimensional heat transfer code can easily be modified to describe heat transfer during the manufacturing of thermoplastic composites.[6] The same energy balances can be used; only properties, heat generation term due to crystallization, and boundary conditions need to be modified. The code can be extended to three-dimensional systems since the grid generation technique and discretization of the differential equation have been established. It is believed, however, that enormous computer time would be required, especially if adaptive grids and moving boundaries are considered.

The grid generation technique can be easily extended to multiple bodies in order to model heat transfer through several parts being simultaneously cured in the autoclave. Local grids would be generated in each part considered as a subsystem. Then a global grid is superimposed over the system in order to transfer information from one subsystem to another without interpolation. Temperature gradients within the autoclave can then be accommodated, and large scale production facilities can be modeled.

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